A Robust T Wave Alternans Detector Based on the GLRT for Laplacian Noise Distribution

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Abstract

T wave alternans (TWA) has been proposed as a marker of cardiac instability and high risk for malignant ventricular arrhythmias and sudden cardiac death. Several algorithms have been used to detect and estimate TWA, such as the spectral method (SM), complex demodulation (CD) or correlation method. In this work, we show that SM and CD methods can be understood as a Generalized Likelihood Ratio Test (GLRT) for TWA episodes in Gaussian noise. However, the noise distribution in ECG recordings is more 'heavy-tailed' than Gaussian noise due to outliers (eg artifacts, impulsive noise, baseline wandering, or ectopic beats). In this paper, we derive a similar GLRT TWA detector and amplitude estimator for Laplacian noise. The resulting estimator is, in fact, a "median filtered complex demodulation". Simulation results suggest that this new approach is more robust than CD or SM when physiological noise is present. The effect is clearer when the noise distribution has heavier tails, as in the case of muscular noise.

1. Introduction

Electrical T-wave alternans (TWA), defined as a consistent fluctuation in the repolarization morphology which repeats on every-other-beat basis, have been documented in a wide range of experimental and clinical situations, such as long QT syndrome, myocardial ischemia and infarction, coronary artery occlusion, Printzmetal angina and several other pathologic conditions.

Although visible TWA is an infrequent phenomenon, in recent years, computerized analysis of digital ECG recordings allowed the identification of subtle and non-visible (microvolt) TWA, much more common than visible TWA. Recently, several studies showed that TWA is related to cardiac instability and high risk for malignant ventricular arrhythmias and sudden cardiac death [1]. Thus the importance of developing robust and sensitive methods for detecting TWA in ECG signals.

Several methods for TWA detection have been proposed. All of them are based on the well-known problem of spectral estimation. The spectral method (SM) [1] used the FFT to analyze the frequency component 0.5 cycles/beat over the aligned ST-T complexes. In the complex demodulation approach (CD) [2], the alternant component in the aligned ST-T complexes is demodulated and low-pass filtered to obtain a continuous beat-to-beat alternans measurement. More recently several global methods have been applied that consider the repolarization as a whole, based on Karhunen-Loève transform [3] and on the correlation with a median beat [4].

In this paper, we derive and evaluate a more robust detector developing a GLRT detector under the assumption of Laplacian noise. In Section 2, we show that the SM and CD approaches can be considered GLRT detectors of TWA in Gaussian noise, we develop the GLRT for Laplacian noise and we describe the simulation study. In Section 3, we show some simulation results and in Section 4, we expose the conclusions of the work.

2. Materials and methods

2.1. GLRT for TWA in Gaussian noise

In all the published methods the ECG signal is reduced to a number of beat-to-beat series related to the ST-T complex of the ECG (usually the amplitude of the ST-T complex at a given sample within the complex). The TWA detection methods are applied to these beat-to-beat series.

Each of these series, can be modelled as:

$$x[n] = a \cdot e[n] \cdot (-1)^n + C + w[n] \;\; n = 0 \cdots N-1, \quad (1)$$

where $e[n]$ is the normalized beat-to-beat shape of the TWA, a and C are unknown constants wich represent the TWA amplitude and the component of the T wave which repeats on every beat. $w[n]$ represents the additive noise, which is usually assumed to be white and Gaussian with pdf $\sim \mathcal{N}(0, \sigma^2)$. If $\sum_{n=0}^{N-1} a \cdot e[n] \cdot (-1)^n = 0$ (eg, if the episode shape is symmetrican and N is even), the term C can be easily cancelled by subtracting the mean of the observed values. The model for the observed detrended series is $y[n] = x[n] - C = a\, e[n]\ (-1)^n + w[n], \;\; n = 0 \cdots N-1$.

If the shape of the episode, $e[n]$, is known, this information can be used to estimate and detect the TWA episodes.

In TWA detection, we have the following hypothesis testing problem:

$$\begin{aligned} \mathcal{H}_0 &: a = 0 \\ \mathcal{H}_1 &: a \neq 0 . \end{aligned} \quad (2)$$

This problem holds the classical linear model. To obtain the GLRT detector [5, Theorem 7.1], we need the maximum likelihood estimator (MLE) of a under $\mathcal{H}_1$:

$$\widehat{a}=\frac{1}{N}\sum_{n=0}^{N-1} y[n]e[n](-1)^n. \quad (3)$$

and the corresponding GLRT is to decide $\mathcal{H}_1$ if

$$T(\mathbf{y}) = \frac{\frac{1}{N}\left(\sum_{n=0}^{N-1} y[n]e[n](-1)^n\right)^2}{\sigma^2} > \gamma, \quad (4)$$

which is the periodogram of the detrended data $y[n]$ windowed by $e[n]$, evaluated at $f = 0.5$. The noise variance in the denominator is suposed to be known. Otherwise, it can be estimated or simply included in an adaptive threshold.

To detect transient episodes beginning at an unknown time within the observed data, the GLRT detector consists on a sliding window approach, applying (4) to each group of *N* correlative samples, and looking for the maximum. This implementation can also be seen as the output of a filter matched to the TWA episode shape.

To detect episodes with a given shape $e[n]$, the GLRT in eq. (4) is esentially the modified periodogram, windowing the data with $e[n]$. In the sliding window version, the test is the square of the output of the filter with impulse response $h[n] = e[-n](-1)^n$ to the input $y[n]$, or equivalently, the result of complex demodulation of the signal $y[n]$ using a low-pass filter with impulse response $g[n] = e[-n]$.

If nothing is known about the TWA shape, it can be considered stationary during the intervals of *N* samples. Then $e[n] = 1$, $n = 0 \cdots N-1$ (a rectangular window) and $T(\mathbf{y})$ is the periodogram of the series divided by the noise power. This GLRT for rectangular-shaped episodes is esentially the SM for TWA detection.

The Gaussian noise model used in this Section, usually assumed because of its mathematical simplicity and justified by the central limit theorem, does not characterize well some types of noise, due to the presence of "noise spikes". In ECG recordings, the presence of artifacts, baseline wandering, or ectopic beats, usually make the noise in the beat-to-beat series to be more "spiky" than Gaussian noise. The Laplacian distribution is an example of a more heavy-tailed statistic (Pearson kurtosis α_4=6) and can model the noise in the TWA series better than the Gaussian one, while still keeping some mathematical tractability.

2.2. GLRT for TWA in Laplacian noise

Now, the model is the same as in (1) but the noise is assumed to be white and with Laplacian pdf:

$$p(w[n]) = \frac{1}{\sqrt{2\sigma^2}} \exp\left(-\sqrt{\frac{2}{\sigma^2}}\,|w[n]|\right), \quad (5)$$

The constant term C in (1) can be again easily cancelled. If we assume a constant episode shape of the TWA within the analysis window, ie $e[n] = 1 \quad n = 0 \cdots N-1$, then, the model for the detrended signal is $y[n] = x[n] - C = a \cdot (-1)^n + w[n] \;\; n = 0 \cdots N-1$.

The multivariate pdf of the detrended data under $\mathcal{H}_1$ for a given *a* is:

$$\begin{aligned} &p(y[0], y[1] \cdots y[N-1];\, a,\, \mathcal{H}_1) \\ &= \left(\tfrac{1}{2\sigma^2}\right)^{\frac{N}{2}} \exp\left(-\sqrt{\tfrac{2}{\sigma^2}} \textstyle\sum_{i=0}^{N-1} |y[n] - a \cdot (-1)^n|\right). \end{aligned} \quad (6)$$

Thus, the MLE of *a*, $\widehat{a}$, is the value that minimizes $J(a) = \sum_{i=0}^{N-1} |y[n] - a \cdot (-1)^n| = \sum_{i=0}^{N-1} |y[n](-1)^n - a|$. It can be shown [5] that

$$\widehat{a} = \text{median}_{n=0}^{N-1}\,(y[n](-1)^n) = z_{med}. \quad (7)$$

The MLE is the median of the demodulated series $z[n] = y[n](-1)^n$. This equation is similar to (3), but computing the median instead of the average. Substituting this value into the logarithm of GLRT, we obtain the following test:

$$T(\mathbf{y}) = 2 \ln L_G(\mathbf{y}) = \sqrt{\frac{8}{\sigma^2}} \sum_{n=0}^{N-1} (|z[n]| - |z[n] - z_{med}|)\,, \quad (8)$$

which can be simplified [5] to

$$T'(\mathbf{y}) = \begin{cases} \sqrt{\frac{32}{\sigma^2}} \sum_{\{n:\, 0 < z_n < z_{med}\}} z_n & \text{if } z_{med} \geq 0 \\ -\sqrt{\frac{32}{\sigma^2}} \sum_{\{n:\, z_{med} < z_n < 0\}} z_n & \text{if } z_{med} < 0 \end{cases}, \quad (9)$$

proportional to the sum of the absolute values of all samples of the demodulated series with values between zero and the median. Again If the noise variance is not known, it can be absorbed by the threshold. A sliding-window implementation is also possible when the position of the episode beginning is unknown. From equations (3) and (9) it is clear that both the GLRT alternans detector and MLE alternans estimator (the output of a median filter) are robust to the presence of outliers in the data series.

2.3. Detectors and estimators

Four TWA detectors were implemented and evaluated in this study: Complex Demodulation (CD), Spectral Method (the sliding window variant) (SM), Median filtered complex

demodulation (MCD) and the GLRT for Laplacian noise). All of them share the same preprocessing stage, including QRS detection, baseline wandering, linear filtering (20th-order equiripple linear phase FIR low-pass filter with transition band between 15 and 30 Hz), ST-T segmentation, alineation and decimation. Thus, the ECG signal is transformed into 17 beat-to-beat series corresponding to samples within the ST-T segment. The tests used for detection and the amplitude estimators are given in Table 1. The rms values of each test over the whole ST-T segment were compared to a threshold to decide if there was or not TWA.

Table 1. *Detection test and amplitude estimator used in the implemented methods.*

	Test	Estimator
CD	$\left\|\sum_{n=0}^{N-1} y[n]h[-n](-1)^n\right\|$	$\frac{1}{N}\sum_{n=0}^{N-1} y[n]e[n](-1)^n$
SM	$\left\|\sum_{n=0}^{N-1} y[n]\,(-1)^n\right\|$	$\frac{1}{N}\sum_{n=0}^{N-1} y[n](-1)^n$
MCD	$\left\|\text{median}\,(y[n]\,(-1)^n)\right\|$	$\text{median}\,(y[n]\,(-1)^n)$
GLRT	eq. (9)	-

2.4. Simulation study

To evaluate the TWA detectors under different noise conditions, we propose the simulation study shown in Figure 1. Clean ECG segments of 128 beats were obtained by repetiting a single beat. Noise and TWA were added to this signal.

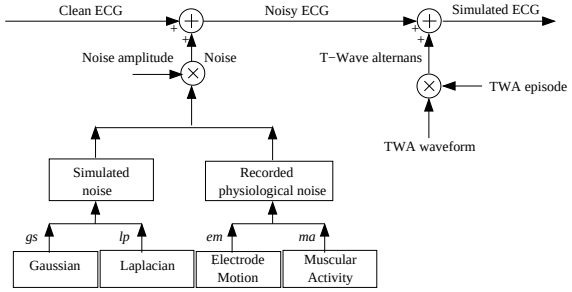

Figure 1. Simulation of ECG signals with T-wave alternans episodes and four noise sources.

Four different noise sources were considered: simulated Gaussian and Laplacian noise and two recordings of physiological noise from the *MIT-BIH Noise Stress Test* database [6]: electrode motion ('em') and muscular activity ('ma'). To compare the effects of the different noise types, the noise were scaled so that the noise level after linear filtering (aproximately the band DC-20Hz) was the desired.

TWA was simulated by adding and subtracting a hanning window to the ST-T complex of the simulated beats. The amplitude of this waveform was modulated beat-by-beat by a trapezoidal episode shape of 40 beats of duration and being 18 beats at its maximum value. The episodes were centered in the 128-beat segment.

The noise type and level and the TWA amplitude were the parameters of the proposed simulation.

3. Results

3.1. Statistics of the noise

The degree of nonGaussianity of a random variable is tipically measured by its kurtosis relative to a Gaussian pdf. We have computed the kurtosis α_4 of the MIT-BIH noise recordings for *em* and *ma* noises, after baseline filtering and applying the same lowpass filter as the one used with the signal. We have consists of two channels of thirty minutes. The total kurtosis of each channel are $\alpha_4^{em,0} = 7.7$, $\alpha_4^{em,1} = 3.5$, $\alpha_4^{ma,0} = 16.8$, $\alpha_4^{ma,1} = 10.0$. Dividing the recordings in 15 2-minute fragments (in the order of the length of the data processed together by the detectors), we obtain a mean kurtosis of 4.4 for *em* and 9.4 for *ma* noise. Clearly, both kinds of noise are more heavy-tailed than normal distribution ($\alpha_4 = 3$), especially, the muscular noise. So, it can be expected a better performance of the TWA detector derived from the Laplacian noise model ($\alpha_4 = 6$) .

3.2. Simulation results

For this study, ECG segments with the described noise types and with noise levels ranging from 0 μV to 100 μV were simulated, with TWA amplitude of 20 μV and also without TWA. For each parameter set, 100 realizations were analyzed with the four detectors, and receiver-operating characteristic (ROC) curve were relating the sensitivity and specificity were obtained sweeping different threshold values. Figure 2 shows the ROC curves of the four detectors with a noise level of 40 μV (i.e., an alternans-to-noise ratio, ANR, of -6 dB).

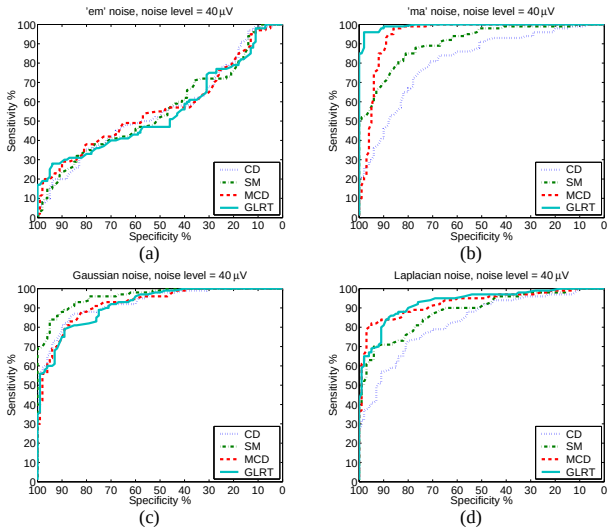

Figure 2. ROC curves for a noise level of 40 μV.

To compare in a simple way the behavior of the detectors when they face with different levels of noise, we summarize the information in the ROC curve with a single paramater, called *S*95 and defined as the sensitivity (%) of the detector using a threshold so that the specificity is 95%. In Figure 3 *S*95 is plotted against the ANR for different noise types and detectors.

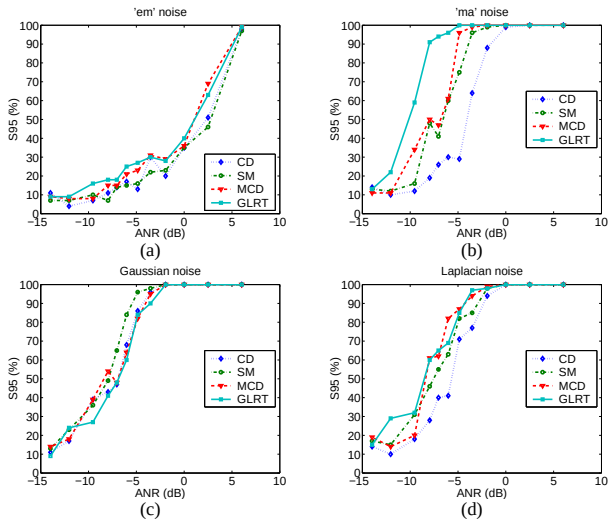

Figure 3. Performance of the detector (*S*95) as a function of the noise level.

The performance observed is similar for all the studied detectors when the noise distribution is Gaussian, as it can be seen in pannel (c). In the case of 'em' noise, the performance is also similar using any approach, but clearly poorer than with the same level of Gaussian noise. When the ECG is contaminated with Laplacian noise, the GLRT and MCD detectors perform slightly better than the classical CD and SM approaches. The clearest differences between the detectors are observed when facing to 'ma' noise, which has the more heavy-tailed noise distribution. Then, the GLRT approach for Laplacian noise surpasses widely the performance of the others: while the GLRT detector obtains obtain *S*=95%, *Se*=95% with *ANR*=-7dB, the CD needs an ANR of -1 dB to get the same performance.

4. Discussion and conclusions

Both the widely-used SM and CD methods for TWA detection can be interpreted as GLRT detectors (and also ML estimators) matched to TWA episodes with a given shape and duration (rectangular in the case of SM and the impulse response of the low-pass filter in CD) under the hypothesis of white Gaussian noise.

A new robust approach for detecting TWA has been proposed, using a signal model including Laplacian noise. As the Laplacian noise distribution is more heavy-tailed than the Gaussian one, the resulting GLRT detector and ML estimator are intrinsecally more insensitive to extreme values. This robustness is expressed mathematically in eq. (7) and (9) by the 'median' function. It is clear that changing arbitrarily any sample in the beat-to-beat series has little effect on both equations.

Numerical results of the simulations show that all the methods perform similarly with Gaussian noise. With Laplacian noise, the GLRT and the median filtered complex demodulation (MCD) seem to have better performance than the classical methods based on a linear combination of the data, but the differences were not so big as expected. On the other hand, when the kurtosis of the noise is even greater (e.g. with 'muscular activity' noise), the performance of the linear methods decays and the proposed GLRT approach makes the difference.

Acknowledgments

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